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Complexity and Cross-Sectional Returns

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Abstract

The paper shows that complexity influences stock returns. By establishing the complexity and resilience measure of the common stock and analyzing the relationship between return, momentum, size, complexity, book-to-market ratio and resilience, three measures (size, complexity and momentum) stand out as the factors that can influence stock returns. Using data from 2009 to 2013, complexity has negative relationship with stock returns. Complexity-based trading strategies can provide more stable returns than size-based and momentum-based strategies. These findings might be exploited in factor investing.

1. Introduction

There are many factors that can influence the cross section of stock returns. Among them, the state-of-the-art ones are size, book-to-market ratio and momentum. Many studies support such factor premiums. Based on these, factor investing is getting more and more popular. People are not satisfied with investing in the whole index or the single stock and begin to invest based on these factor measures. From the financial crisis, we all know that financial systems are threatened by excessive complexity. Excessive complexity can lead to many disasters like plane crash(Zolli 2012), financial crisis and the bankruptcy of a company. Those complex structure financial products and "too big to fail" institutions make the systems vulnerable to external shocks. In this paper, two complexity-related measures are introduced as the new risk factors. The famous heuristic Occam's razor states that given two equally options, the one with the lowest complexity should be chosen. This is contrast to the basis behind capital asset pricing model. In CAPM, given two portfolios with the same expected returns, the one with the lower risk should be chosen.

There are few studies about the relationship between complexity and stock returns. The reason is that the definition of complexity is difficult. Through Ontonix,

it is possible to measure the complexity of a system. High complexity implies a capacity to deliver surprising behavior. Stocks with higher complexity might be more fragile and offer lower returns. This implies a negative relationship between complexity and stock returns. According to Marczyk(2011), ‘‘highly complex systems are difficult to manage and can quickly run out of hand. ’’ (p. 29) A highly complex system is also less efficient than simple ones. Consequently, complexity is a crucial parameter for every system, including the common stock. Another risk factor that is introduced into the study is resilience. Resilience is the measure of the system’s ability to resist extreme shocks.

This paper provides a systematic study of the relationship between complexity and equity return. Using the CRSP/Compustat Merged Database, complexity is measured for the components of S&P 100 index. There exists significant cross-sectional variation in complexity across different stocks. Three quantitative criteria are used to compare complexity factor with other factors which can influence equity returns. The three criteria are correlation between the factor and the stock return, t statistics of the factor in the multiple regression and the performance of factor-based strategy.

Different factor investing strategies are also compared for the whole market and for the factor-based submarket. In the large cap subgroup and high complexity subgroup, the complexity-based trading strategy can offer superior returns. This suggests that highly complex companies are fragile and vulnerable to external shocks.

Then, returns on different factor-based trading strategies are calculated in the test period. Complexity-based trading strategy performs best by considering both return and standard deviation.

Some limitations of the study need to be highlighted. Due to the difficulty of measuring the complexity of the common stock, we only test the cross-sectional data in a single period from 2012 to 2013. It’s still unknown whether this relationship still exists in different time periods, different assets as well as different countries.

The remainder of the paper is organized as follows. Section 2 will discuss related literature. Section 3 will be devoted to measuring the complexity of the common stock. Section 4 provides the theoretical basis for the relationship between complexity and

stock returns. Section 5, 6 and 7 provide the holistic comparison of different factors that can influence equity returns. Section 8 concludes.

2. Related literature

Only few studies exist on the role complexity plays in the financial market. Marczyk(2011) introduces complexity-based rating method, which is different from traditional default-based rating method and is more adapted to the globalized and chaotic economy. Mariotti(2008) shows that complexity adds costs and kills profits in the business world. Meucci(2010) proposes an entropy pooling approach for trading and portfolio management.

However, empirical asset pricing is a central problem in finance. Three factors are identified by Fama and French(1993) on the return of stocks. Besides a market factor, firm size and book-to-market equity are discovered by them as two factors which play an important role in determining asset returns. Blitz, Falkenstein and Vilet(2013) summaries the cause and effect of 'volatility effect', which is that investing in low-volatility stocks can achieve higher risk-adjusted returns. Goyal(2011) provides a survey on the huge amount of evidence in understanding the cross-sectional difference in asset returns. During the survey, there is no complexity-related evidence. Recently, there are some innovative studies on asset market. Preis, Moat, and Stanley(2013) analyzes the relationship between Google trends data and the return of Dow Jones Industrial Average Index(DJIA). Besides that DeMiguel, Garlappi and Uppal(2009) analyzes the financial data and concludes there is no significant gain out of sample by optimal diversification(like CAPM). It shows that the traditional asset pricing model is under attack and new paradigm is needed.

3. Measuring the complexity of the common stock

1) Dataset

Through Wharton Research Data Services(WRDS), it is possible to use the CRSP/Compustat database. This database provides the fundmental data as well as market data for 27,000 stocks from NASDAQ, New York Stock Exchange(NYSE) and NYSE MKT. It covers all S&P 500 components and most stocks listed in the US security market. The financial statement data as well as the market price data are all

available from the database. The accuracy of the data is crucial for estimating the complexity of the common stock.

Five risk factors are analyzed in this paper: size, book-to-market ratio, momentum, complexity and resilience. The measure of size, book-to-market ratio and momentum are obtained directly from the database. The measure of complexity and resilience is calculated by a model-free algorithm provided by Ontonix using the data from the database. The universe of stocks used in this study is all S&P 100 components since this is a preliminary analysis.

2) Complexity measures

The definition of complexity is presented in this part. Complexity is not a well-defined concept. There are lots of discussions about complexity and its role in financial crisis, system design and corporate management. But it's hard to see a quantitative measure of the complexity. Ontonix is the world's first company to provide the measure of complexity rationally.

Marczyk(2013) provides a detailed definition of the complexity.

> Complexity is a natural property of every system. Its importance is similar to that of energy. The complexity of a system described by a vector {x} of N components is defined as a function of structure and entropy.
>
> $$C = f(\mathbf{S} \circ \mathbf{E})$$
>
> Where S represents an N × N adjacency matrix, E is an N × N entropy matrix, ○ is the Hadamard matrix product operator and f is a norm operator. The adjacency matrix indicates the correlations between the components of {x}. The adjacency matrix is determined via a multi-dimensional algorithm which determines if entry S_{ij} is 0 or 1. This establishes the structure of the system. The intensity of the dependencies, the so-called generalized correlation, is computed based on entropy. Entropy measures how crisp (or fuzzy) the dependencies between the elements of {x} are. In essence, it quantifies the amount of disorder within the system. The huge advantage of this model-free approach is that it is independent of numerical conditioning of the data and its ability to identify the existence of structures where conventional methods fail. Once the entropy matrix and the adjacency matrix have been obtained, one may compute the complexity of a given system as the following matrix norm:
>
> $$C = \left\| \mathbf{S} \circ \mathbf{E} \right\|$$
>
> A fundamental property of systems related to complexity is the so-called critical complexity, CU, which corresponds to the upper bound of the complexity metric. Critical complexity may be defined formally using the above expression,
>
> $$CU = \left\| \mathbf{S} \circ \mathbf{E}_{max} \right\|$$

where E_{max} is the entropy matrix in which the entries correspond to a situation of maximum sustainable disorder within the system. In a similar fashion, the lower bound of complexity, CL, may be computed as $CL = \| S \circ E_{min} \|$. In proximity of the lower complexity bound, a given system functions in a deterministic structure-dominated fashion. In proximity of the upper complexity bound the system in question is uncertainty-dominated and relationships between the various components of {x} are fuzzy and therefore characterized by very low generalized correlations.

A measure of system resilience may now be defined as follows:

$$R = f(CL; C; CU)$$

where CL, C and CU represent, respectively, the lower complexity bound, the current system complexity and the upper complexity bound. The function f in the above equation is a second-order polynomial function such that:

$$\text{if } C = CL \rightarrow R = 100\%, \text{ if } C = CU \rightarrow R = 0\%$$

(Marczyk, 2013, p3-4)

For measuring the complexity of a common stock, the key issue is to describe a common stock as a system of parameters. 32 parameters are selected to cover market data, index data, corresponding risk-free rate, dividend data, balance sheet data and income statement data. 12 quarters' data of these 32 parameters are used for calculating the complexity of a common stock. This measure of complexity is suitable for the long term. But by using intraday data, high frequency measure can also be obtained using different parameters.

4. Complexity of the common stock

The complexity of the common stock for 99 of the S&P 100 components is measured using 12 quarters' data from 2009 Q1 to 2011 Q4. Only one stock Mondelez International is excluded due to spin-off. The measuring process is done through the website www.rate-a-business.com. The common stock template in the website is used for the analysis. We can also get interactive business structure maps after performing the analysis. Some examples of the stock data used as the input for the analysis can be seen in the appendix. Table 1 shows the mean, standard deviation and other summary statistics for the complexity measure.

Complexity	
Mean	15.35
Standard Error	0.36
Median	15.31
Mode	15.49
Standard Deviation	3.57
Sample Variance	12.74
Kurtosis	-0.06
Skewness	0.39
Range	17.4
Minimum	9.32
Maximum	26.72
Count	99

Table 1: Descriptive statistics of the complexity measure for S&P 100 components

Comparing complexity estimates across stocks, Target Corporation(TGT) has the lowest complexity, which is consistent with its business of operating mainstream discount stores across the United States. The company with the highest complexity is EMC corporation. It can be explained by its business of data storage, virtualization and cloud computing. Besides that the top three companies(EMC, Apple and Amazon) with the highest complexity are all technology companies. This is a really volative industry filled with opportunities as well as risks. Many of the high-tech companies in 1980 disappear today. The standard deviation of 3.57 also reflects the cross-sectional difference in common stock complexities. It's nonnegligible but not very significant when this value is compared with the mean complexity of 15.35 and median complexity of 15.31.

5. Comparison of different risk factors for the whole sample

1) Background information for three tests

Due to the high uncertainty and the danger in the prediction of equity returns, we don't intend to explain the equity return by one or a combination of several variables. It's well known that psychological effect must be taken into consideration for analyzing the market. Instead, we'd like to see which variables can help provide information in the understanding of the cross-sectional difference in common stock returns. Three different kinds of criteria will be used to evaluate the effectiveness and relative importance of different risk factors. The three criteria: correlation between the

factor and the stock return, t statistics of the factor in the multiple regression and the performance of factor-based strategy. Based on the three criteria, we can establish a thorough evaluation of the importance and effectiveness of different risk factors in the specific time period.

We analyze five risk factors. Size and book-to-market measures are identified by FAMA and FRENCH(1992). Momentum measure is offered by JEGADEESH and TITMAN(1993). The importance of the price information is also confirmed in the field of experimental finance. The complexity and resilience measure are new potential risk factors introduced in this article.

We explain the measure of momentum, complexity and resilience. Momentum is defined as one year past return. For measuring the complexity and resilience, we use 32 components to describe the common stock(as provided by the common stock template in the appendix). The past three years' data are used to describe every component. Using these data, we can measure complexity and resilience using OntoSpace™. OntoSpace™ is a quantitative complexity management system developed by Ontonix. The test period is from 02.04.2012 to 01.04.2013. It needs to be noticed that there are no major shocks like Lehman Brothers' bankruptcy in this period. The stock return is calculated based on the adjusted closed price of the stock in 02.04.2012 and 01.04.2013. The bid-ask spread and the problem of liquidity can be neglected due to the return period of 1 year. During this period, the stock with the highest return in the sample is Gilead Sciences (GILD) with the return of 96.72%. The stock with the lowest return in the sample is Apple (AAPL) with the return of -29.64%. By comparison, S&P 500 goes up 10.09% during this period. Besides that, Target Corporation (TGT) has the return of 20.5% while EMC Corporation (EMC) has the return of -20.67%. If you long the least complexity stock and short the highest complexity stock, a significant return can be obtained. This also reflects the role complexity plays in determining stock returns. We use the data of 02.04.2012 to measure size, book-to-market ratio, and momentum and determine the components of the test stocks. Due to the gap between accounting data and market data, we use the 12 quarterly data(from 2009 Q1 to 2011 Q4) to measure complexity and resilience. And the test sample is 99 components of the S&P 100 index. Mondelez International is excluded because the spin-off from Kraft Foods makes the data uncomparable. Due

to the restriction of the computing power, we use only 99 stocks for one single period to perform the test. It can be seen like the first phase in the FDA drug approval.

2) Test 1

In table 2, five risk factors are compared via their correlation with stock return. Two measures (book-to-market ratio and resilience) have almost no correlation with stock returns. Size and complexity have negative relationship with stock return. And momentum has positive correlation with the stock return. The ranking of the relative importance of the three nontrivial risk factors are size, complexity and momentum.

In figure 1, we can see more clearly the relationship between complexity and 1 year return. The slightly negative trend can be seen clearly and the negative relationship is more significant in the right end of the horizontal axis.

Correlation between size and 1 year return	-0.18301
Correlation between book-to-market and 1 year return	0.004104
Correlation between complexity and 1 year return	-0.18105
Correlation between resilience and 1 year return	-0.01487
Correlation between momentum and 1 year retrun	0.112042

Table 2: Correlation between risk factors and stock return

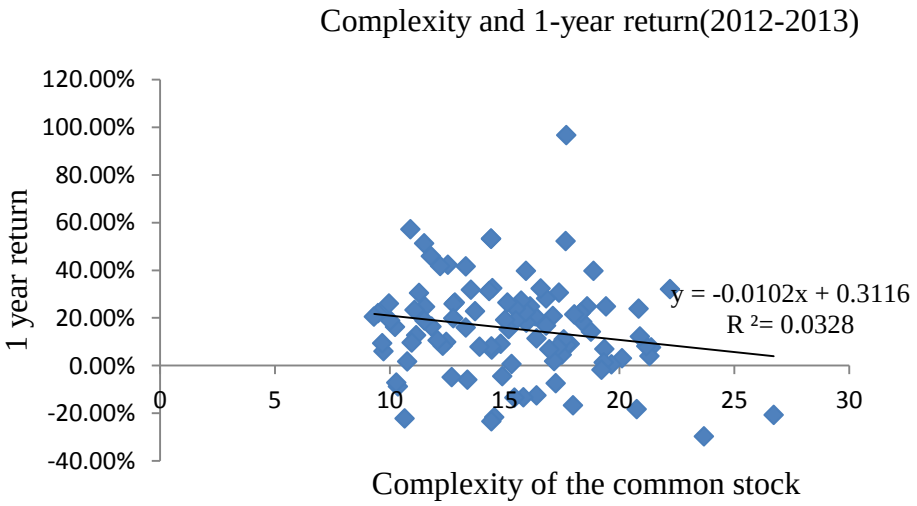

Figure 1: Scatter plot for complexity of the common stock and 1 year return of the stock

3) Test 2

Next, we evaluate different risk factors via multiple regression. The return variable is regressed against different risk factors for several models and the one with the highest adjusted R-square value is chosen. As can be seen in table 3, in the regression with all five risk factors, only three factors(size, complexity and momentum) have the absolue value of t statistic larger than 1. This suggests attempting regression with these three risk factors (size, complexity and momentum)

Risk factors	*t Stat*
Intercept	0.557
Size	-1.942
book-to-market	0.119
Complexity	-1.448
Resilience	0.778
Momentum	1.578

Table 3: t-statistic in the multiple regression including all five risk factors

The result of the regression is presented in table 4. All three factors have t statistics with absolute value larger than 1.5. In particular the t statistic of complexity doesn't change much and is significantly negative, supporting the negative relationship between complexity and cross-sectional returns. Repeating the regression for other combinations of factors also supports the important role of these three factors. In table 5, the multiple regression with these three factors is the best model according to the comparison of the adjusted r-square value. The low value of adjusted r-square value confirms the unpredictability of stock returns. Investors should not make investment decisions only based on several risk factors.

Risk factors	*t Stat*
Intercept	3.586
Size	-1.905
Momentum	1.825
Complexity	-1.516

Table 4: t-statistic in the multiple regression including three risk factors

Factors in the model	Adjusted R Square
All factors	0.044
Size, Momentum, Complexity and Resilience	0.054
Size, Momentum and Complexity	0.057
Size, Momentum	0.044
Momentum, Complexity	0.031
Size, Complexity	0.034

Table 5: Adjusted r-square value for different regression models

Finally we find size, momentum and complexity have significant importance in determining the cross-sectional returns. This finding is robust to the specification of the regression model. The relative importance of the three factors according to the ranking of the absolute value of the t statistic in the three factor regression model is size, momentum and complexity.

6. Comparison of different risk factors for the high complexity group

The reason of comparing different risk factors for the high complexity group(with the complexity value no less than the median complexity value) is that the complexity measure is significantly important as an early warning for turbulent companies. This can be seen from figure 2. The strength of relationship can be compared with another two important measures(size and book-to-market ratio) by comparing figure 2 with figure 3 and figure 4.

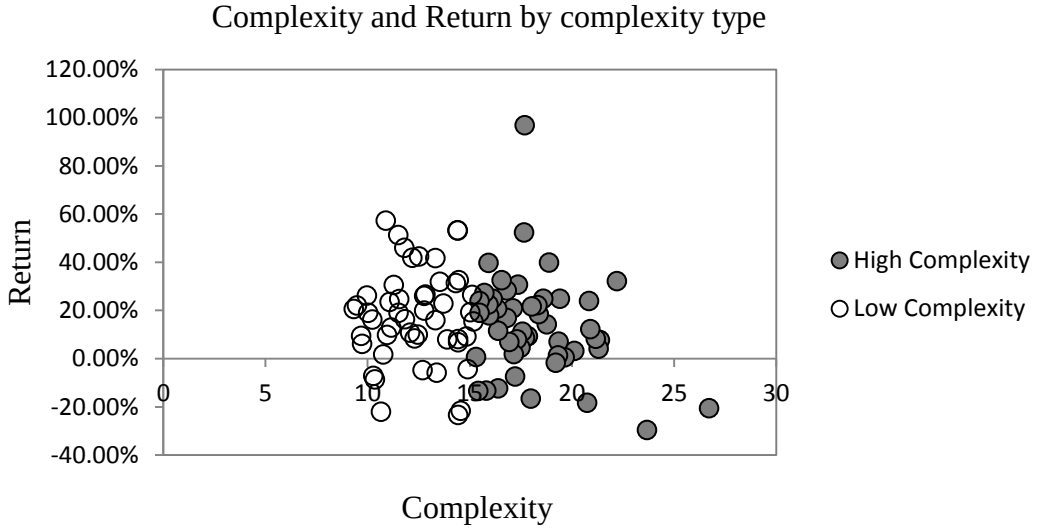

Figure 2: Relationship between complexity and 1-year return by different complexity groups

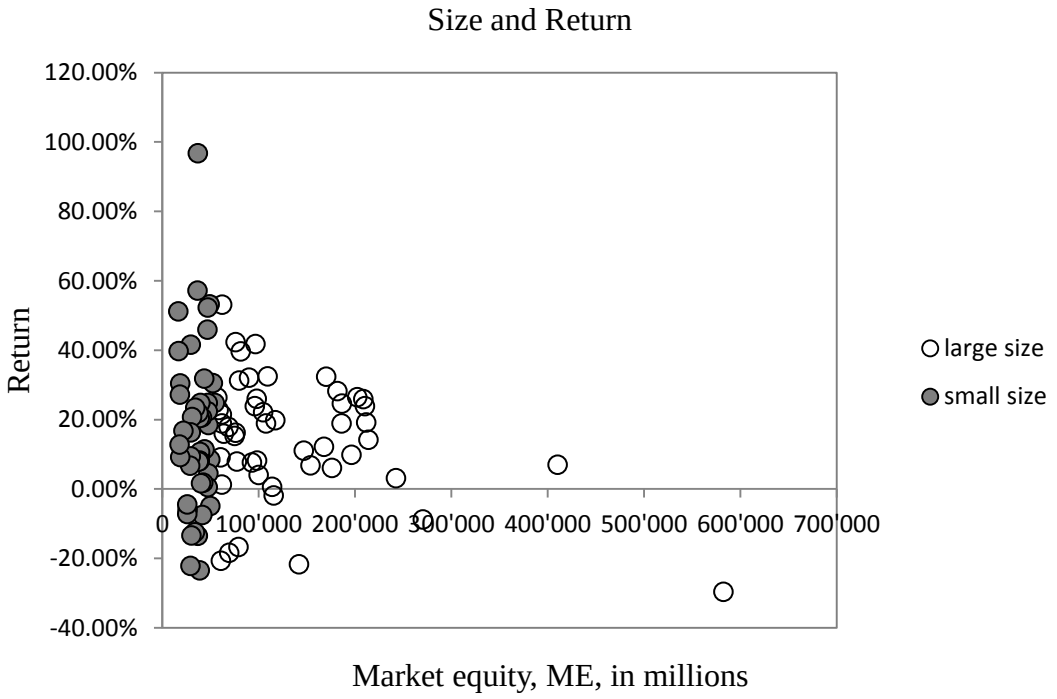

Figure 3: Relationship between size and 1-year return by different size groups

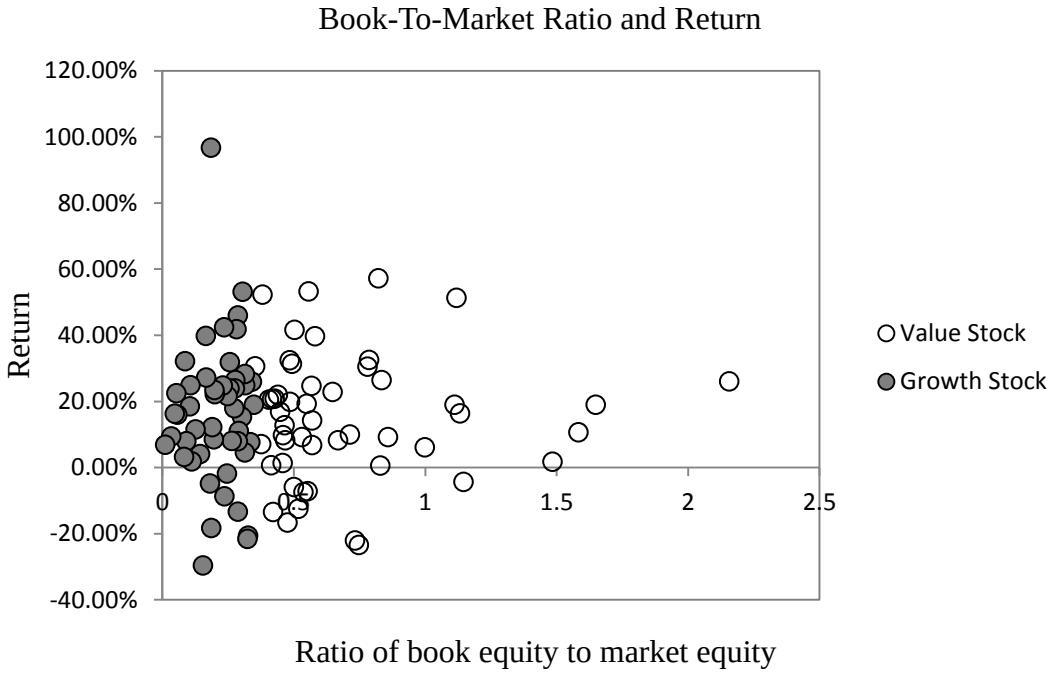

Figure 4: Relationship between size and book-to-market ratio by different book-to-market ratio groups

For the common stock with the low complexity value or simple operations, the measure is not so important. And the complexity measure is very important for complex and fragile companies. This can be confirmed by the table 6 and figure 5. In table 6, five factors' correlations with the return are compared. We can see that complexity ranks first and the ranking of the relative importance of the three nontrivial risk factors are complexity, size and resilience. We can also see the increase of correlation between complexity and 1 year return. By contrast the change of sign for the book-to-market ratio shows the instability of this measure.

Correlation between size and 1 year return	-0.24868
Correlation between book-to-market and 1 year return	-0.08589
Correlation between complexity and 1 year return	-0.28101
Correlation between resilience and 1 year return	-0.12086
Correlation between momentum and 1 year retrun	0.023852

Table 6: Correlation between risk factors and stock return for the high complexity group

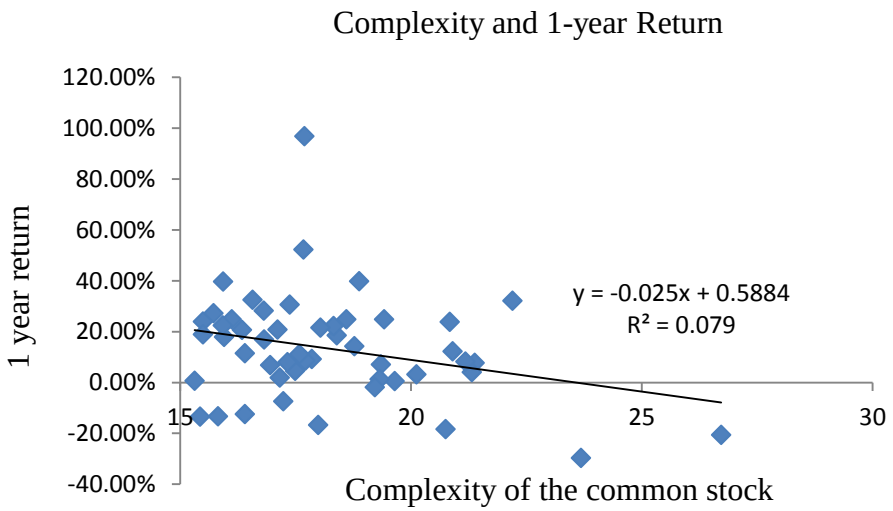

Figure 5: Scatter plot for complexity of the common stock and 1 year return of the stock for the high complexity group

Results from multiple regression also confirm this. From Table 7, the ranking of the factors has changed from size, momentum and complexity to complexity, book-to-market ratio and momentum. These results are consistent with the role complexity plays as an early warning for fragility of the business, which is finally reflected as the highly negative return of the corresponding stock. For instance, the increase of 1 for the complexity value leads to 2 percentage drop in 1 year return on average.

Risk factors	Coefficient	*t Stat*
Intercept	0.580	1.123
Size	0.002	0.009
book-to-market	0.000	-1.033
Complexity	-0.021	-1.444
Resilience	0.008	0.011
Momentum	-0.132	-0.745

Table 7: Coefficient and t-statistics in the multiple regression including all risk factors for the high complexity group

7. Comparison of different factor-based trading strategies

Finally we compare the five factors based on the performance of factor-based strategy. This analysis is more relevant because the comparison basis is tradable returns. In constructing the portfolio, we divide all stocks into two subgroups with same number of stocks based on the ranking of the risk factor. Then we calculate the difference between the return of the group with large values in the risk factor and the return of the group with small values in the risk factor. To decide the weighting of each stock in the subgroup, we choose the naive 1/N portfolio strategy. This strategy is validated by DeMiguel, Garlappi and Uppal(2009). They find that among 14 models examined (including CAPM and CAPM-derived model), no one performs better than 1/N rule out of sample consistently. Table 8 shows the performance of these subgroups. We can rank these factors based on the difference of return between the two factor-based subgroups. The ranking result is momentum, complexity, resilience, size and book-to-market ratio.

Subgroup Type	Return
Large Cap Subgroup	14.35%
Small Cap Subgroup	16.53%
Difference between the two size-based subgroup	-2.18%
Value Stock Subgroup	14.66%
Growth Stock Subgroup	16.14%
Difference between the two book-to-market ratio based subgroup	-1.48%
High Complexity Subgroup	13.59%
Low Complexity Subgroup	17.82%
Difference between the two complexity-based subgroup	-4.22%
High Resilience Subgroup	14.08%
Low Resilience Subgroup	16.81%
Difference between the two resilience-based subgroup	-2.74%
High Momentum Subgroup	19.63%
Low Momentum Subgroup	11.13%
Difference between the two momentum-based subgroup	8.50%

Table 8: Average return for different subgroups

Based on these three tests, we can see that size, complexity and momentum stand out in this period. We would like to explore better the interaction of different factors. We divide the components into different subgroups. Then within each subgroup we compare the average return for different subgroups based on different risk factors.

From table 9, we can firstly see the large uncertainty in stock returns. Then we can see in the large cap group and high complexity group, complexity-based portfolio performs well. This is consistent with the idea that the fragility of highly complex and large business can lead to low returns. It also shows complexity is an early warning sign for the crisis. The measure and monitor of complexity is especially important for those large cap and high complexity companies. For investors, complexity-based trading strategy performs much better in these groups. And avoid investing in those highly complex and fragile companies is a good advice. These companies are very vulnerable to external shocks. From table 10, we can do a summary of the return difference of different factors in different subgroups. We can see if we only consider average return, momentum factor performs best. If we consider both average return and standard deviation, then the complexity factor performs best. In summary, complexity is an important factor for stock returns during the sample period.

Return	large cap	small cap	high complexity	low complexity	high momentum	low momentum
Large Cap Subgroup	11.66%	16.32%	11.51%	18.72%	13.06%	14.30%
Small Cap Subgroup	16.72%	17.10%	16.19%	16.23%	26.65%	8.80%
Difference based on size	-5.06%	-0.78%	-4.68%	2.49%	-13.59%	5.50%
High Complexity Subgroup	8.92%	17.25%	8.17%	17.88%	18.05%	9.02%
Low Complexity Subgroup	19.39%	16.21%	15.55%	17.75%	22.60%	14.09%
Difference based on complexity	-10.46%	1.04%	-7.38%	0.13%	-4.55%	-5.07%
High Momentum Subgroup	14.92%	29.01%	18.49%	24.38%	13.30%	16.37%
Low Momentum Subgroup	15.25%	4.45%	7.61%	10.56%	26.44%	5.46%
Difference based on momentum	-0.33%	24.56%	10.88%	13.82%	-13.14%	10.91%

Table 9: Average return for different subgroups within the specific subgroup

	Average Return	Standard deviation
Size	-2.69%	6.72%
Complexity	-4.38%	4.39%
Momentum	7.78%	12.97%

Table 10: Summary of the performance of size-based strategy (the return is 1 year return)

8. Conclusions

Our results show that among five plausible measures, three measures (size, complexity and momentum) stand out as the factors that can influence stock returns. Using data from 2009 to 2013, we find that the rise of complexity has negative relationship with stock returns. We also find the complexity-based trading strategy can provide more stable returns than size-based strategy and momentum-based strategy. That means investors may require a premium for high complexity stocks. However, it remains to be investigated whether these findings are specific to the sample period. Meanwhile the unpredictability of stock returns is also significant based on the study. Investors should be cautious about making investment decisions based on several risk factors.

Excessive complexity implies fragility. That might be the reason that complexity has negative relationship with equity returns. This is confirmed by the fact that the complexity-based strategy performs better in the large cap group and the high complexity group.

In this work, we find that besides two well-known measures (size and momentum), complexity provides significant information relating to the stock returns. The study opens up new doors into factor investing as well as the measure of complexity in the bond and foreign exchange market.

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Appendix 1. 32 parameters for the Common Stock Template

Market Capitalization
P/E Ratio
Return on S&P 500
Return on Nasdaq
Return on FTSE 100
Inflation
Treasury return
Return on stock
Average Trading Volume
Close price
High price
Low price
Dividend
Assets
Cash and cash equivalents
Short-term investments
Total Inventories
Accounts receivable
Total current assets
Total Intangible assets
Debt in current liabilities
Goodwill
Income taxes
Total preferred stock
Total cost of goods sold
Total depreciation and amortization
Research and development expense
Selling, General and Administrative expenses
Foreign exchange income(Loss)
Non-operating income(expense)
Net income
Total long term investments

Appendix 2. Examples of stock data used for measuring complexity at www.rate-a-business.com

PARAMETER	2009 Q1	2009 Q2	2009 Q3	2009 Q4	2010 Q1	2010 Q2	2010 Q3	2010 Q4	2011 Q1	2011 Q2	2011 Q3	2011 Q4
Market Capitalization	109903.77	133400.35	157152.22	197012.28	180499.55	141766.85	167850.04	190843.16	189011.24	163392.62	166747.81	209849.68
P/E Ratio	25.44	29.24	32.03	30.38	25.80	19.32	21.35	22.58	22.78	18.26	17.55	21.70
Return on S&P 500	5.68%	13.14%	4.93%	3.64%	10.51%	-7.17%	7.41%	8.69%	6.03%	-5.23%	-3.02%	4.72%
Return on Nasdaq	16.32%	15.21%	3.37%	5.00%	14.62%	-8.39%	11.21%	7.68%	6.42%	-4.08%	-2.61%	4.82%
Return on FTSE 100	2.27%	8.59%	9.46%	2.85%	7.03%	-5.32%	7.93%	3.31%	3.53%	-4.20%	-4.66%	2.48%
Inflation	0.40%	0.75%	0.76%	0.45%	0.14%	0.32%	0.54%	0.80%	0.88%	0.58%	0.26%	0.62%
Treasury return	0.05%	0.05%	0.03%	0.01%	0.04%	0.03%	0.04%	0.04%	0.03%	0.01%	0.00%	0.00%
Return on stock	13.14%	21.13%	17.61%	25.03%	-8.53%	-21.54%	18.17%	12.97%	-1.21%	-13.70%	1.71%	25.41%
Average Trading Volume	5744167	3564867	2814967	2596567	3842633	3567533	2963600	3000233	2722000	2935900	3980200	3179333
Close price	348.06	421.59	495.85	619.98	567.12	444.95	525.79	593.97	586.76	506.38	515.04	645.9
High price	381	447.34	507	625.99	629.51	597.84	536.849	630.85	642.96	595.19	627.4992	646.76
Low price	282.75	340.61	395.9801	482.6	520	444.72	433.63	518.8501	551.2801	473.02	490.86	480.596
Dividend	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Assets	33513.029	35158.76	37702.845	40496.778	42871	48045	53342	57851	59960	64861	69088	72574
Cash and cash equivalents	10426.291	11911.351	12087.115	10197.588	9192	10713	11257	13630	12415	10320	10630	9983
Short-term investments	7358.642	7432.655	9907.276	14287.187	17322	19346	22998	21345	24260	28798	31930	34643
Total Inventories	0	0	0	0	0	0	0	0	0	0	0	0
Accounts receivable	2543.105	2847.755	2879.604	3201.715	3084	3617	3844	5002	4818	5843	5861	6172
Total current assets	22080.802	23834.853	26353.544	29166.958	31132	35161	39447	41562	43310	46442	50042	52758
Total Intangible assets	5740.659	5693.299	5672.465	5677.503	5912	6692	7092	7300	7324	8058	8463	8924
Debt in current liabilities	0	0	0	0	0	0	2122	3465	3216	1217	1218	1218
Goodwill	4830.315	4836.913	4849.217	4902.565	5122	5788	6099	6256	6308	6677	6989	7346
Income taxes	466.973	371.631	427.566	594.571	551	594	547	599	594	580	630	785
Total preferred stock	0	0	0	0	0	0	0	0	0	0	0	0
Total cost of goods sold	1702.175	1732.489	1851.838	2051.383	2125	2131	2209	2571	2540	2718	2889	3204
Total depreciation and amortization	399.329	375.482	374.402	357.017	327	336	343	375	396	453	489	499
Research and development expense	641.643	707.626	757.524	736.234	818	898	995	1051	1226	1234	1404	1298
Selling, General and Administrative expenses	1497.595	1541.032	1644.893	1784.442	1835	1988	2188	2512	2843	2974	3285	3373
Foreign exchange income(Loss)	-90.92	-83.701	-73.856	-11.301	-105	-106	-69	-75	-128	-72	-24	-155
Non operating income(expense)	6.21	-17.718	-7.177	87.688	18	69	168	160	100	217	323	2
Net income	1422.828	1484.545	1638.975	1974.1	1955	1840	2167	2543	1798	2506	2729	2704
Total long term investments	100.999	93.925	110.372	128.977	154	377	485	523	666	893	891	790

Table 11: Data of Google used for measuring the complexity

PARAMETER	2009 Q1	2009 Q2	2009 Q3	2009 Q4	2010 Q1	2010 Q2	2010 Q3	2010 Q4	2011 Q1	2011 Q2	2011 Q3	2011 Q4
Market Capitalization	47043.94	47041.41	58465.40	64693.11	67060.71	59359.25	53910.90	50180.87	61104.60	58188.65	58854.02	69810.45
P/E Ratio	36.10	31.77	22.29	25.88	25.01	19.38	18.52	16.68	16.43	17.09	16.61	18.73
Return on S&P 500	5.68%	13.14%	4.93%	3.64%	10.51%	-7.17%	7.41%	8.69%	6.03%	-5.23%	-3.02%	4.72%
Return on Nasdaq	16.32%	15.21%	3.37%	5.00%	14.62%	-8.39%	11.21%	7.68%	6.42%	-4.08%	-2.61%	4.82%
Return on FTSE 100	2.27%	8.59%	9.46%	2.85%	7.03%	-5.32%	7.93%	3.31%	3.53%	-4.20%	-4.66%	2.48%
Inflation	0.40%	0.75%	0.76%	0.45%	0.14%	0.32%	0.54%	0.80%	0.88%	0.58%	0.26%	0.62%
Treasury return	0.05%	0.05%	0.03%	0.01%	0.04%	0.03%	0.04%	0.04%	0.03%	0.01%	0.00%	0.00%
Return on stock	6.21%	12.17%	11.16%	26.75%	4.24%	-22.16%	5.14%	-5.04%	4.81%	14.67%	1.90%	18.72%
Average Trading Volume	9974333	10499200	5724900	7239367	5170067	10099067	6596900	6427000	6524000	6460267	6360033	4432967
Close price	55.6	62.26	69.11	87.46	91.03	70.75	74.26	70.38	73.62	84.26	85.72	101.53
High price	57.98	71.24	74.99	89.69	93.63	97.192	77.8	81.75	77.08	87.36	94.75	103.45
Low price	41.78	53.63	58	66.5452	80.54	68.29	64.9	66.5	67.51	73.11	76.11	81.71
Dividend	0.11	0.11	0.11	0.13	0.13	0.13	0.13	0.15	0.15	0.15	0.15	0.22
Assets	31964	32624	32281	31715	32068	32696	33408	33989	34422	34447	34760	35768
Cash and cash equivalents	3363	4200	4617	4160	4560	5205	3867	3684	3512	3600	2127	1942
Short-term investments	1783	1341	2361	2323	2292	2819	2949	3750	4064	4044	5059	6059
Total Inventories	0	0	0	0	0	0	0	0	0	0	0	0
Accounts receivable	1386	2441	1184	1272	1168	917	1018	1033	935	956	1084	1138
Total current assets	8225	9562	9241	8753	9115	9850	8734	9355	9606	9516	9190	9978
Total Intangible assets	21096	21096	21091	21091	21091	21091	22925	22910	23052	23121	23104	23122
Debt in current liabilities	52	52	12	12	12	12	12	12	12	13	0	0
Goodwill	10213	10213	10208	10208	10208	10208	11447	11447	11588	11668	11668	11668
Income taxes	352	568	349	445	401	406	422	498	497	539	476	590
Total preferred stock	0	0	0	0	0	0	0	0	0	0	0	0
Total cost of goods sold	706	763	916	724	773	829	917	805	786	903	943	849
Total depreciation and amortization	56	57	61	62	62	63	78	67	70	74	77	80
Research and development expense	0	0	0	0	0	0	0	0	0	0	0	0
Selling, General and Administrative expenses	0	0	0	0	0	0	0	0	0	0	0	0
Foreign exchange income(Loss)	0	0	0	0	0	0	0	0	0	0	0	0
Non operating income(expense)	36	32	19	7	19	-2	18	16	-6	113	-8	9
Net income	536	729	514	763	713	716	774	884	881	1005	880	1029
Total long term investments	856	231	270	260	252	233	138	126	100	108	811	899

Table 12: Data of Visa Inc. used for measuring the complexity

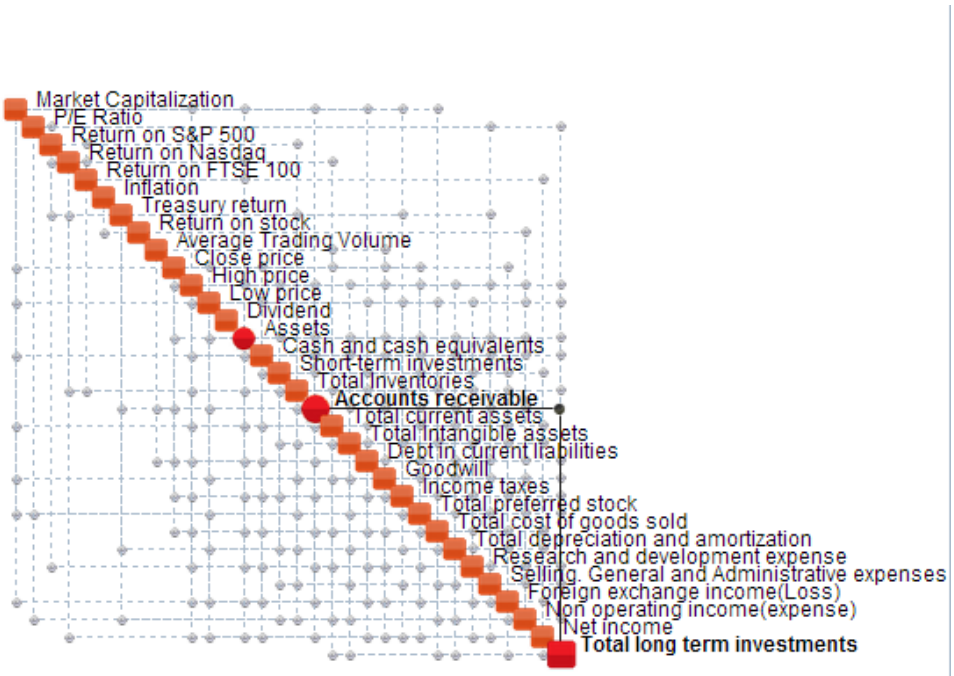
PARAMETER	2009 Q1	2009 Q2	2009 Q3	2009 Q4	2010 Q1	2010 Q2	2010 Q3	2010 Q4	2011 Q1	2011 Q2	2011 Q3	2011 Q4
Market Capitalization	23082.80	29950.91	33701.38	33714.98	35741.61	32639.39	34144.10	35354.83	38484.60	38261.24	30449.52	35200.36
P/E Ratio	12.37	16.79	17.55	16.75	15.87	13.77	14.74	14.30	16.08	15.68	11.40	13.34
Return on S&P 500	5.68%	13.14%	4.93%	3.64%	10.51%	-7.17%	7.41%	8.69%	6.03%	-5.23%	-3.02%	4.72%
Return on Nasdaq	16.32%	15.21%	3.37%	5.00%	14.62%	-8.39%	11.21%	7.68%	6.42%	-4.08%	-2.61%	4.82%
Return on FTSE 100	2.27%	8.59%	9.46%	2.85%	7.03%	-5.32%	7.93%	3.31%	3.53%	-4.20%	-4.66%	2.48%
Inflation	0.40%	0.75%	0.76%	0.45%	0.14%	0.32%	0.54%	0.80%	0.88%	0.58%	0.26%	0.62%
Treasury return	0.05%	0.05%	0.03%	0.01%	0.04%	0.03%	0.04%	0.04%	0.03%	0.01%	0.00%	0.00%
Return on stock	-12.76%	31.56%	15.03%	1.83%	8.14%	-6.90%	6.78%	5.72%	11.68%	2.54%	-16.95%	21.48%
Average Trading Volume	10797667	13419467	8318600	7542167	9143133	10418133	6699733	7973633	9600133	7536100	10170933	7621833
Close price	19.3	25.19	28.78	29.14	31.27	28.91	30.65	32.17	35.7	36.37	29.97	36.14
High price	33.4503	26.83	30.485	32.85	31.75	34.07	33.23	32.9	38.34	38.62	37.26	36.69
Low price	17.81	18.98	23.34	28.42	26.43	28.43	27.99	29.42	31.44	34.23	27.62	28.43
Dividend	0.19	0.19	0.19	0.19	0.21	0.21	0.21	0.21	0.24	0.24	0.24	0.24
Assets	70829	70920	71266	65730	66066	65030	65220	66524	65742	66628	66890	67801
Cash and cash equivalents	7115	7009	7126	4800	5167	4238	4009	3663	3029	3520	3245	3476
Short-term investments	0	0	0	0	0	0	0	0	0	0	0	0
Total Inventories	2050	1840	1892	1779	1890	1860	1945	1920	1948	1899	1999	1890
Accounts receivable	4674	4847	4833	5111	5143	5328	5309	6413	5854	5902	5742	6922
Total current assets	15287	15243	15252	13007	13393	12631	12334	13138	11903	12439	12082	13432
Total Intangible assets	43572	43390	43279	40699	40213	40068	40101	40313	40265	40302	40241	40066
Debt in current liabilities	2080	2087	2090	59	260	34	34	26	28	29	22	23
Goodwill	32357	32064	31978	29795	29758	29697	29826	29994	30012	30080	30095	30029
Income taxes	227	299	320	307	389	317	221	421	331	313	431	409
Total preferred stock	0	0	0	0	0	0	0	0	0	0	0	0
Total cost of goods sold	3193	3169	3250	3955	3189	3429	3361	4370	3564	3880	3648	4566
Total depreciation and amortization	242	231	240	235	232	236	222	248	231	230	228	233
Research and development expense	0	0	0	0	0	0	0	0	0	0	0	0
Selling, General and Administrative expenses	1494	1452	1444	1653	1477	1504	1465	1716	1583	1617	1557	1660
Foreign exchange income(Loss)	0	0	0	0	0	0	0	0	0	0	0	0
Non operating income(expense)	18	36	-64	-36	30	27	0	61	19	6	0	-182
Net income	660	524	662	622	725	562	522	769	653	638	822	773
Total long term investments	1900	2127	2081	2032	1743	1679	1656	1796	1983	1998	1925	1820

Table 13: Data of Times Warner used for measuring the complexity

PARAMETER	2009 Q1	2009 Q2	2009 Q3	2009 Q4	2010 Q1	2010 Q2	2010 Q3	2010 Q4	2011 Q1	2011 Q2	2011 Q3	2011 Q4
Market Capitalization	22948.23	26488.83	34758.57	35856.14	37083.17	37579.36	41846.64	47385.73	54649.01	56889.26	42818.70	44133.09
P/E Ratio	19.32	25.19	36.26	32.96	30.07	26.14	26.04	26.02	28.87	28.70	20.78	19.58
Return on S&P 500	5.68%	13.14%	4.93%	3.64%	10.51%	-7.17%	7.41%	8.69%	6.03%	-5.23%	-3.02%	4.72%
Return on Nasdaq	16.32%	15.21%	3.37%	5.00%	14.62%	-8.39%	11.21%	7.68%	6.42%	-4.08%	-2.61%	4.82%
Return on FTSE 100	2.27%	8.59%	9.46%	2.85%	7.03%	-5.32%	7.93%	3.31%	3.53%	-4.20%	-4.66%	2.48%
Inflation	0.40%	0.75%	0.76%	0.45%	0.14%	0.32%	0.54%	0.80%	0.88%	0.58%	0.26%	0.62%
Treasury return	0.05%	0.05%	0.03%	0.01%	0.04%	0.03%	0.04%	0.04%	0.03%	0.01%	0.00%	0.00%
Return on stock	8.88%	14.91%	30.08%	2.52%	3.26%	1.44%	10.98%	12.75%	15.98%	3.73%	-23.81%	2.62%
Average Trading Volume	29650000	27701700	23041333	21145133	25640900	26466967	27908633	21486433	25254167	19590267	25079100	22769767
Close price	11.4	13.1	17.04	17.47	18.04	18.3	20.31	22.9	26.56	27.55	20.99	21.54
High price	12.59	13.735	17.48	18.44	19.04	20	21.83	23.2	27.5901	28.73	28.25	25.13
Low price	9.61	10.91	12.31	16.12	16.45	17.1	17.87	19.39	22.84	25.39	19.84	19.99
Dividend	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Assets	23902.099	24487.332	25551.744	26812.003	27467.963	28011.451	28926.908	30833.284	31187.432	32331.862	32430.671	34268.179
Cash and cash equivalents	6323.452	6422.76	5518.359	6302.499	6256.626	5784.812	4940.592	4119.138	4102.332	3896.369	3125.61	4531.036
Short-term investments	928.277	835.49	617.535	392.839	641.576	1015.143	1712.41	1256.175	1491.454	1436.29	1635.533	1786.987
Total Inventories	820.504	773.528	810.925	886.289	851.501	813.756	838.622	856.405	921.004	1005.69	1104.143	1009.968
Accounts receivable	1700.096	1855.986	1864.121	2108.575	1735.043	1976.311	2120.679	2589.423	2379.175	2620.448	2650.392	2964.499
Total current assets	10596.161	10686.262	9628.565	10538.302	10375.125	10546.303	10596.233	9783.322	10172.458	10233.41	9947.723	11582.683
Total Intangible assets	7042.473	7109.565	9222.725	10376.175	9402.851	9434.792	9787.321	13962.917	11790.048	12108.156	12153.651	14586.485
Debt in current liabilities	0	0	0	0	0	0	0	3214.771	3244.134	3271.865	3280.957	3304.974
Goodwill	7042.473	7109.565	9222.725	9210.376	9402.851	9434.792	9787.321	11772.65	11790.048	12108.156	12153.651	12154.97
Income taxes	37.815	38.045	20.602	156.313	95.653	136.976	148.663	257.005	121.639	172.731	171.086	174.929
Total preferred stock	0	0	0	0	0	0	0	235.229	205.866	175.907	144.862	119.325
Total cost of goods sold	1210.813	1258.964	1282.162	1440.258	1390.005	1378.88	1428.236	1617.199	1569.295	1548.948	1554.86	1677.645
Total depreciation and amortization	255.798	253.268	281.977	282.092	281.58	284.859	296.525	304.586	339.272	349.803	359.076	373.447
Research and development expense	383.293	397.881	422.092	424.243	434.933	477.725	483.264	492.093	502.108	538.891	548.021	560.767
Selling, General and Administrative expenses	1399.307	1441.518	1583.883	1689.452	1682.094	1751.357	1819.313	1975.641	1998.039	2114.58	2160.935	2355.615
Foreign exchange income(Loss)	-28.4	-28.4	-28.4	-21.2	-21.2	-21.2	-21.2	-4.5	-4.5	-4.5	-4.5	-12.8
Non operating income(expense)	29.086	31.36	40.306	16.226	22.511	34.233	34.74	11.558	-4.947	10.343	-27.506	-56.593
Net income	194.069	205.232	298.18	390.596	372.704	426.216	472.516	628.559	477.148	546.494	605.649	832.046
Total long term investments	2533.237	2765.946	2291.672	2692.323	3254.641	3539.361	3860.062	4285.199	3876.569	4180.205	4499.373	4889.614

Table 14: Data of EMC Corp used for measuring the complexity

Appendix 3. Examples of complexity maps and complexity profiles for the common stock



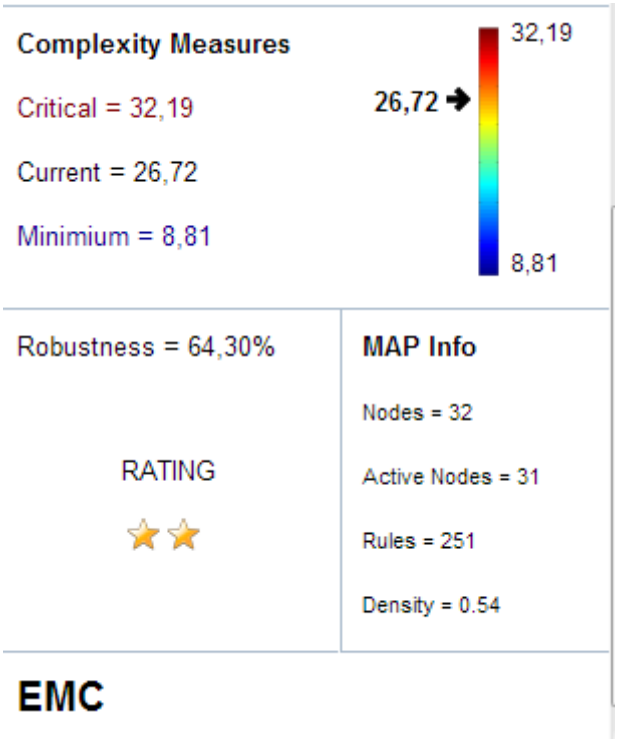


Figure 6: Complexity map of the EMC Corporation

Var. #	Variable Name	% Contribution
11	High price	6,73
14	Assets	6,54
32	Total long term investments	5,71
27	Research and development expense	5,68
20	Total Intangible assets	5,64
26	Total depreciation and amortization	5,14
15	Cash and cash equivalents	5,12
18	Accounts receivable	5,04
29	Foreign exchange income(Loss)	5,00
25	Total cost of goods sold	4,97
22	Goodwill	4,88
31	Net income	4,75
23	Income taxes	4,68
28	Selling. General and Administrative expenses	4,57
19	Total current assets	4,53

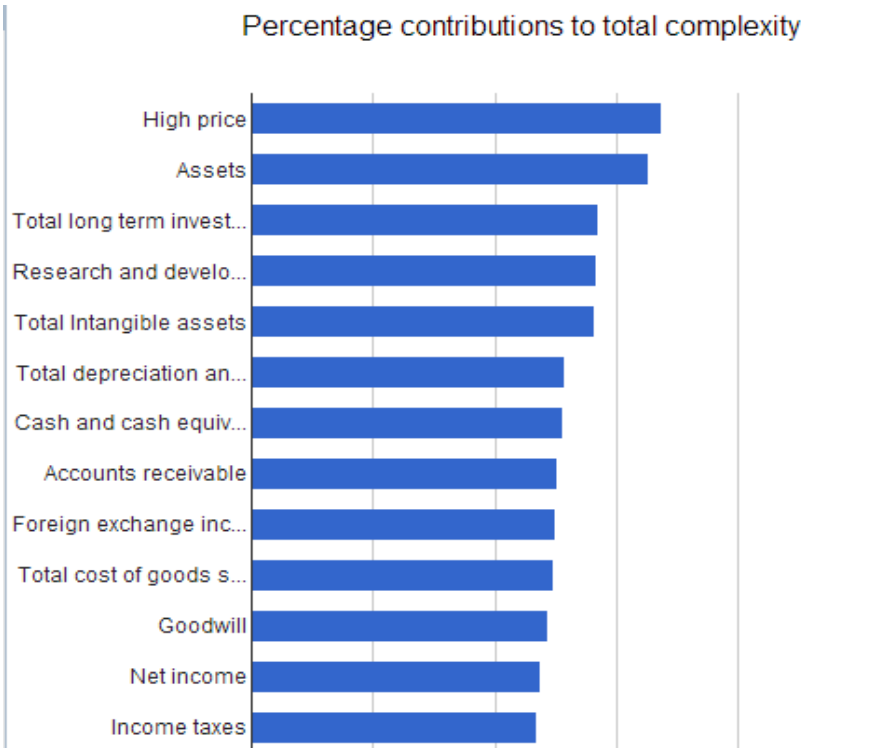


Figure 7: Complexity profile of the EMC Corporation

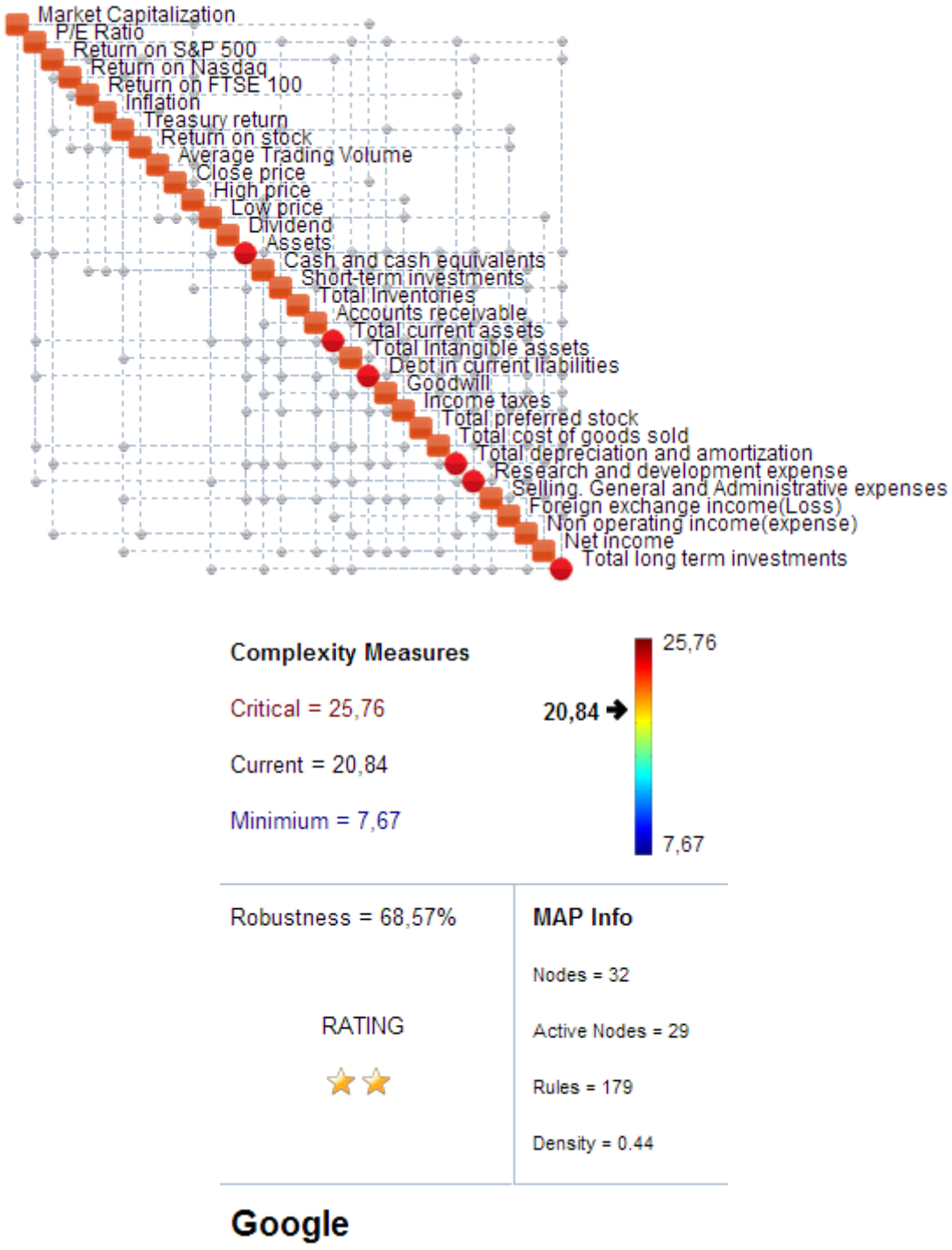


Figure 8: Complexity map of Google

Var. #	Variable Name	% Contribution
27	Research and development expense	7,17
16	Short-term investments	6,90
26	Total depreciation and amortization	6,83
25	Total cost of goods sold	6,80
15	Cash and cash equivalents	6,64
19	Total current assets	6,56
14	Assets	6,56
32	Total long term investments	6,53
21	Debt in current liabilities	6,39
22	Goodwill	6,38
20	Total Intangible assets	6,17
18	Accounts receivable	5,63
28	Selling. General and Administrative expenses	5,62
2	P/E Ratio	3,24
31	Net income	3,20

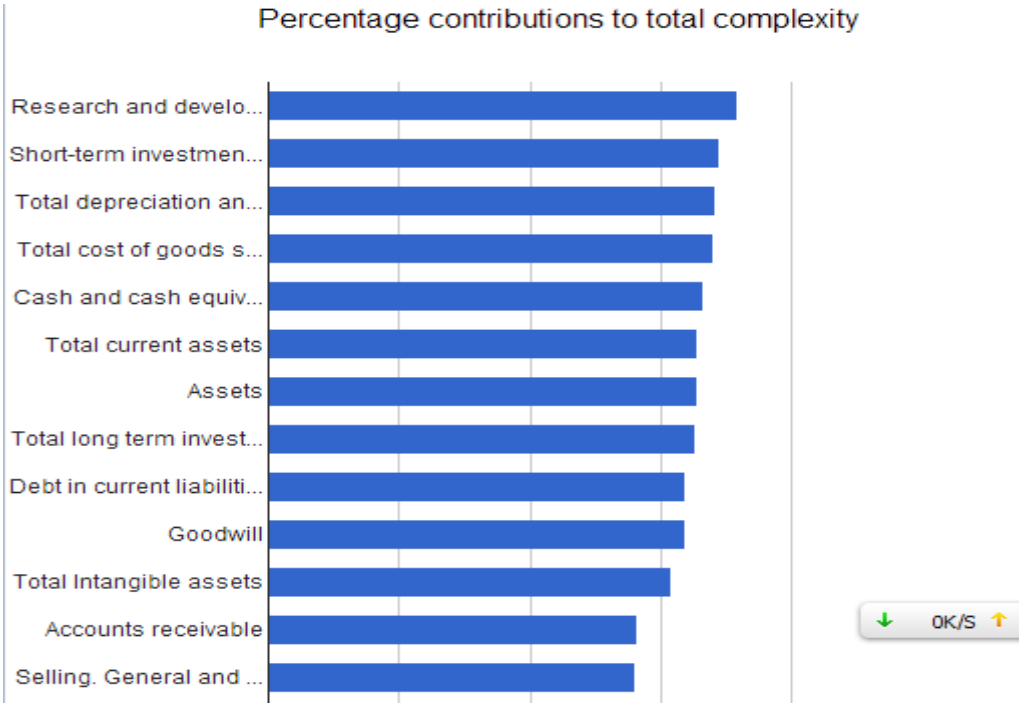


Figure 9: Complexity profile of Google